T-TEST

T Test procedure compares the means of two groups or (one-sample) compares the means of a group with a constant.

Two Independent Samples

Notation

The following notation is used unless otherwise stated:

X_{ki}	Value for *i*th case of group *k*
w_{ki}	Weight for *i*th case of group *k*
n_k	Number of cases in group *k*
W_k	Sum of weights of cases in group *k*

Basic Statistics

Means

$$\bar{X}_k = \frac{\sum_{i=1}^{n_k} X_{ki} w_{ki}}{W_k} \qquad k = 1, 2$$

Variances

$$S_k^2 = \frac{\sum_{i=1}^{n_k} X_{ki}^2 w_{ki} - \left(\sum_{i=1}^{n_k} X_{ki} w_{ki} \right)^2 \Big/ W_k}{(W_k - 1)}$$

Standard Errors of the Mean

$$SEM_k = S_k \big/ \sqrt{W_k}$$

Differences of the Means for Groups 1 and 2

$$D = \overline{X}_1 - \overline{X}_2$$

Unpooled (Separate Variance) Standard Error of the Difference

$$S_D = \sqrt{\frac{S_1^2}{W_1} + \frac{S_2^2}{W_2}}$$

The 95% confidence interval for mean difference is

$$D \pm t_{df'} S_D$$

where $t_{df'}$ is the upper 2.5% critical value for the t distribution with df' degrees of freedom.

Pooled Standard Error of the Difference

$$S'_D = S_p \sqrt{\frac{1}{W_1} + \frac{1}{W_2}}$$

where the pooled estimate of the variance is

$$S_p^2 = \frac{(W_1 - 1)S_1^2 + (W_2 - 1)S_2^2}{W_1 + W_2 - 2}$$

The 95% confidence interval for mean difference

$$D \pm t_{df} S'_D$$

where *df* is defined in the following.

The *t* Statistics for Equality of Means

Separate Variance

$$t = D / S_D$$
$$df' = \frac{1}{Z_1 + Z_2}$$

where

$$Z_k = \left(\frac{S_k^2 / W_k}{S_1^2 / W_1 + S_2^2 / W_2} \right)^2 \Big/ (W_k - 1)$$

Pooled Variance

$$t' = D / S'_D$$
$$df = W_1 + W_2 - 2$$

The two-tailed significance levels are obtained from the *t* distribution separately for each of the computer *t* values.

The Test for Equality of Variances

The Levene statistic is used and defined as

$$L = \frac{(W-2)\sum_{k=1}^{2} W_k (\overline{Z}_k - \overline{Z})^2}{\sum_{k=1}^{2}\sum_{i=1}^{n_k} w_{ki} (Z_{ki} - \overline{Z}_k)^2}$$

where

$$Z_{ki} = \left| X_{ki} - \overline{X}_k \right|$$

$$\overline{Z}_k = \frac{\sum_{i=1}^{n_k} w_{ki} Z_{ki}}{W_k}$$

$$\overline{Z} = \frac{\sum_{k=1}^{2} W_k \overline{Z}_k}{W_1 + W_2}$$

The *t* Test for Paired Samples

Notation

The following notation is used unless otherwise stated:

X_i	Value of variable *X* for case *i*
Y_i	Value of variable *Y* for case *i*
w_i	Weight for case *i*
W	Sum of the weights
N	Number of cases

Basic Statistics

Means

$$\overline{X} = \sum_{i=1}^{N} w_i X_i / W$$

$$\overline{Y} = \sum_{i=1}^{N} w_i Y_i / W$$

Variances

$$S_X^2 = \frac{\sum_{i=1}^{N} w_i X_i^2 - \left(\sum_{i=1}^{N} w_i X_i \right)^2 \Big/ W}{W - 1}$$

Similarly for S_Y^2.

Covariance between *X* and *Y*

$$S_{XY} = \frac{1}{W-1} \sum_{k=1}^{N} \left(X_k Y_k w_k - \left(\sum_{k=1}^{N} w_k X_k \right) \left(\sum_{k=1}^{N} w_k Y_k \right) \Big/ W \right)$$

Difference of the Means

$$D = \overline{X} - \overline{Y}$$

Standard Error of the Difference

$$S_D = \sqrt{\left(S_X^2 + S_Y^2 - 2 S_{XY} \right) \Big/ W}$$

t statistic for Equality of Means

$$t = D/S_D$$

with $(W-1)$ degrees of freedom. A two-tailed significance level is printed.

95% Confidence Interval for Mean Difference

$$D \pm t_{W-1} S_D$$

Correlation Coefficient between X and Y

$$r = \frac{S_{XY}}{S_X S_Y}$$

The two-tailed significance level is based on

$$t = r\sqrt{\frac{W-2}{1-r^2}}$$

with $(W-2)$ degrees of freedom.

One-Sample *t* Test

Notation

The following notation is used unless otherwise stated:

N	Number of cases
X_i	Value of variable X for case i ($i = 1, \ldots, N$)
w_i	Weight for case i ($i = 1, \ldots, N$). The weights must be positive.
v	Test value

Basic Statistics

Mean

$$\overline{X} = \frac{1}{W}\sum_{i=1}^{N} w_i X_i$$

where $W = \sum_{i=1}^{N} w_i$ is the sum of the weights.

Variance

$$S_X^2 = \frac{1}{W-1}\sum_{i=1}^{N} w_i\left(X_i - \overline{X}\right)^2$$

Standard Deviation

$$S_X = \sqrt{S_X^2}$$

Standard Error of the Mean

$$S_{\overline{X}} = S_X / \sqrt{W}$$

Mean Difference

$$D = \overline{X} - v$$

The *t* value

$$t = D / S_{\overline{X}}$$

with $(W-1)$ degrees of freedom. A two-tailed significance level is printed.

100p% Confidence Interval for the Mean Difference $(0 < p < 1)$

$$\text{CI} = D \pm t_{W-1,(p+1)/2} S_{\overline{X}}$$

where $t_{W-1,(p+1)/2}$ is the $100\big((p+1)/2\big)\%$ percentile of a Student's *t* distribution with $(W-1)$ degrees of freedom.

References

Blalock, H. M. 1972. *Social statistics*. New York: McGraw-Hill.