

# FREQUENCIES

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If the absolute value of any observation is greater than  $10^{13}$ , no calculations are done. For sorting of the observations, see Appendix 6. For information on percentiles for grouped data, see Appendix 8.

## Notation

The following notation is used throughout this chapter unless otherwise stated:

$X_k$	Value of the variable for case $k$
$w_k$	Weight for case $k$
$NV$	Number of distinct values the variable assumes
$N$	Number of cases
$W$	Sum of weights of the cases

## Basic Statistics

The values are sorted into ascending order and the following calculated:

### Sum of Weights of Cases Having Each Value of $X$

$$f_j = \sum_{i=1}^N w_i k_i \quad j = 1, 2, \dots, NV$$

where

$$k_i = \begin{cases} 1 & \text{if } X_i = X_j \\ 0 & \text{otherwise} \end{cases}$$

where  $X_j$  is the  $j$ th largest distinct value of  $X$ .

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### Relative Frequency (Percentage) for each Value of $X$

$$Rf_j = \left( \frac{f_j}{W'} \right) \times 100$$

where

$$W' = \sum_{i=1}^{NV} f_i \quad (\text{sum over all categories including those declared as missing values})$$

### Adjusted Frequency (Percentage)

$$Af_j = \left( \frac{f_j}{W} \right) \times 100$$

where

$$W = \sum_{i=1}^{NV} f_i k_i \quad (\text{sum over nonmissing categories})$$

and

$$k_i = \begin{cases} 0 & \text{if } X_i \text{ has been declared missing} \\ 1 & \text{otherwise} \end{cases}$$

For all  $X_j$  declared missing, an adjusted frequency is not printed.

### Cumulative Frequency (Percentage)

$$Cf_j = \sum_{i=1}^j f_i$$

**Minimum**

$$\min_k X_k$$

**Maximum**

$$\max_k X_k$$

**Mode**

Value of  $X_j$  which has the largest observed frequency. If several are tied, the smallest value is selected.

**Range**

Maximum – Minimum

**The  $p$ th percentile**

Find the first score interval ( $x_2$ ) containing more than  $tp$  cases.

$$p\text{th percentile} = \begin{cases} x_2 & \text{if } tp - cp_1 \geq 100/W \\ \left\{ 1 - [(W+1)p/100 - cc_1] \right\} x_1 & \text{if } tp - cp_1 < 100/W \\ + [(W+1)p/100 - cc_1] x_2 & \end{cases}$$

where

$$tp = (W+1)p/W$$

$$cp_1 < tp < cp_2$$

$x_1$  and  $x_2$  are the values corresponding to  $cp_1$  and  $cp_2$ , respectively

$cc_1$  is the cumulative frequency up to  $x_1$

$cp_1$  is the cumulative percent up to  $x_1$

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### Mean

$$\bar{X} = \frac{\sum_{j=1}^{NV} f_j X_j}{W}$$

Moments about the mean are calculated as:

$$M_j = \sum_{i=1}^{NV} f_i (X_i - \bar{X})^j \quad j = 2, 3, 4$$

### Variance

$$S^2 = \frac{M_2}{(W-1)}$$

### Standard Deviation

$$S = \sqrt{S^2}$$

### Standard Error of the Mean

$$SEM = \frac{S}{\sqrt{W}}$$

Skewness (computed if  $W \geq 3$  and  $S^2 > 0$ ) (Bliss, 1967, p. 144)

$$g_1 = \frac{WM_3}{(W-1)(W-2)S^3} \quad se(g_1) = \sqrt{\frac{6W(W-1)}{(W-2)(W+1)(W+3)}}$$

Kurtosis (computed if  $W \geq 4$  and  $S^2 > 0$ )

$$g_2 = \frac{W(W+1)M_4 - 3(W-1)M_2^2}{(W-1)(W-2)(W-3)S^4} \quad se(g_2) = \sqrt{\frac{4(W^2-1)se(g_1)^2}{(W-3)(W+5)}}$$

## References

Blalock, H. M. 1972. *Social statistics*. New York: McGraw-Hill.

Bliss, C. I. 1967. *Statistics in biology*, Volume 1. New York: McGraw-Hill.