EXSMOOTH

EXSMOOTH produces one period ahead forecasts for different models.

Notation

The following notation is used throughout this chapter unless otherwise stated:

X_t	Observed series, $t = 1, \ldots, n$
$\hat{X}_t$	Forecast of one period ahead from time t
p	Number of periods
k	Number of complete cycles $([n/p])$
e_t	tth residual $\left(X_t - \hat{X}_{t-1}\right)$
S_0	Initial value for series
T_0	Initial value for trend
$I_{1-p}, \ldots, I_0$	Initial values for seasonal factors
m_l	Mean for the lth cycle, $\sum_{i=(l-1)p+1}^{lp} X_i / p$

Please note the following points:

- $I_{1-p}, \ldots, I_0$ are obtained from the SEASON procedure with MA = EQUAL if p is even; otherwise MA = CENTERED is used for both multiplicative and additive models.
- The index for the fitted series starts with zero.
- The value saved in the FIT variable for the tth case is $\hat{X}_{t-1}$.

Models

No Trend, No Seasonality Model

$$X_t = b + \varepsilon_t$$

Initial value

$$S_0 = \overline{X}$$

then

$$\hat{X}_0 = S_0, \quad e_1 = X_1 - \hat{X}_0$$

$$S_t = S_{t-1} + \alpha e_t$$

$$\hat{X}_t = S_t$$

No Trend, Additive Seasonality Model

$$X_t = b + I_t + \varepsilon_t$$

Initial value

$$S_0 = \frac{\sum_{i=1}^{k} m_i}{k}$$

then

$$\hat{X}_0 = S_0 + I_{1-p}$$

$$e_1 = X_1 - \hat{X}_0$$

$$S_t = S_{t-1} + \alpha e_t$$

$$I_t = I_{t-p} + \delta(1-\alpha)e_t$$

$$\hat{X}_t = S_t + I_{t-p+1}$$

No Trend, Multiplicative Seasonality Model

$$X_t = bI_t + \varepsilon_t$$

Initial value

$$S_0 = \frac{\sum_{i=1}^{k} m_i}{k}$$

then

$$\hat{X}_0 = S_0 I_{1-p}$$

$$e_1 = X_1 - \hat{X}_0$$

$$S_t = S_{t-1} + \alpha e_t / I_{t-p}$$

$$I_t = I_{t-p} + \delta(1-\alpha) e_t / S_t$$

$$\hat{X}_t = S_t I_{t-p+1}$$

Linear Trend, No Seasonality Model

$$X_t = b_0 + b_1 t + \varepsilon_t$$

Initial values

$$T_0 = \frac{X_n - X_1}{n-1}$$
$$S_0 = X_1 - \frac{1}{2} T_0$$

then

$$\hat{X}_0 = S_0 + T_0$$

$$e_1 = X_1 - \hat{X}_0$$

$$S_t = S_{t-1} + T_{t-1} + \alpha e_t$$

$$T_t = T_{t-1} + \alpha\gamma e_t$$

$$\hat{X}_t = S_t + T_t$$

Linear Trend, Additive Seasonality Model

$$X_t = b_0 + b_1 t + I_t + \varepsilon_t$$

Initial values

$$T_0 = \frac{m_k - m_1}{(k-1)p}$$

$$S_0 = X_1 - \frac{p}{2} T_0$$

then

$$\hat{X}_0 = S_0 + T_0 + I_{1-p}$$

$$S_t = S_{t-1} + T_{t-1} + \alpha e_t$$

$$T_t = T_{t-1} + \alpha\gamma e_t$$

$$I_t = I_{t-p} + \delta(1-\alpha)e_t$$

$$\hat{X}_t = S_t + T_t + I_{t-p+1}$$

Linear Trend, Multiplicative Seasonality Model

$$X_t = (b_0 + b_1 t) I_t + \varepsilon_t$$

Initial values

$$T_0 = \frac{m_k - m_1}{(k-1)p}$$

$$S_0 = m_1 - \frac{p}{2} T_0$$

then

$$\hat{X}_0 = (S_0 + T_0)I_{1-p}$$

$$S_t = S_{t-1} + T_{t-1} + \alpha\left(e_t / I_{t-p}\right)$$

$$T_t = T_{t-1} + \alpha\gamma\left(e_t / I_{t-p}\right)$$

$$I_t = I_{t-p} + \delta(1-\alpha)\left(e_t / S_t\right)$$

$$\hat{X}_t = (S_t + T_t)I_{t-p+1}$$

Exponential Trend, No Season Model

$$X_t = b_0 b_1^t + \varepsilon_t$$

Initial values

$$T_0 = \exp\left\{\ln X_2 - \ln X_1\right\} = \frac{X_2}{X_1}$$

$$S_0 = \exp\left\{\ln X_1 - \frac{1}{2}\ln T_0\right\} = \frac{X_1}{\sqrt{T_0}}$$

then

$$\hat{X}_0 = S_0 T_0$$

$$S_t = S_{t-1}T_{t-1} + \alpha e_t$$

$$T_t = T_{t-1} + \alpha\gamma e_t / S_{t-1}$$

$$\hat{X}_t = S_t T_t$$

Exponential Trend, Additive Seasonal Model

$$X_t = b_0 b_1^t + I_t + \varepsilon_t$$

Initial values

$$T_0 = \exp\{(\ln m_2 - \ln m_1)/p\}$$

$$S_0 = \exp\left\{\ln m_1 - \frac{p}{2}\ln T_0\right\}$$

then

$$\hat{X}_0 = S_0 T_0 + I_{1-p}$$

$$S_t = S_{t-1}T_{t-1} + \alpha e_t$$

$$T_t = T_{t-1} + \alpha\gamma e_t / S_{t-1}$$

$$I_t = I_{t-p} + \delta(1-\alpha)e_t$$

$$\hat{X}_t = S_t T_t + I_{t-p+1}$$

Exponential Trend, Multiplicative Seasonality Model

$$X_t = (b_0 b_1^t) I_t + \varepsilon_t$$

Initial values

$$T_0 = \exp\{(\ln m_2 - \ln m_1)/(k-1)\}$$

$$S_0 = \exp\left\{\ln m_1 - \frac{p}{2}\ln T_0\right\}$$

then

$$\hat{X}_0 = (S_0 T_0) I_{1-p}$$

$$S_t = S_{t-1} T_{t-1} + \alpha e_t / I_{t-p}$$

$$T_t = T_{t-1} + \alpha\gamma e_t / (I_{t-p} S_{t-1})$$

$$I_t = I_{t-p} + \delta(1-\alpha) e_t / S_t$$

$$\hat{X}_t = (S_t T_t) I_{t-p+1}$$

Damped Trend, No Seasonality Model

$$X_t = b_0 + \phi b_1 t + \varepsilon_t$$

Initial values

$$T_0 = \frac{X_n - X_1}{(n-1)\phi}$$

$$S_0 = X_1 - \frac{1}{2} T_0$$

then

$$\hat{X}_0 = S_0 + \phi T_0$$

$$S_t = S_{t-1} + \phi T_{t-1} + \alpha e_t$$

$$T_t = \phi T_{t-1} + \alpha\gamma e_t$$

$$\hat{X}_t = S_t + \phi T_t$$

Damped Trend, Additive Seasonality Model

$$X_t = b_0 + \phi b_1 t + I_t + \varepsilon_t$$

Initial values

$$T_0 = \frac{m_k - m_1}{(k-1)p\phi}$$

$$S_0 = m_1 - \frac{p}{2} T_0$$

then

$$\hat{X}_0 = S_0 + \phi T_0 + I_{1-p}$$

$$S_t = S_{t-1} + \phi T_{t-1} + \alpha(2-\alpha)e_t$$

$$T_t = \phi T_{t-1} + \alpha(\alpha - \phi + 1)e_t$$

$$I_t = I_{t-p} + \delta[1 - \alpha(2-\alpha)]e_t$$

$$\hat{X}_t = S_t + \phi T_t + I_{t-p+1}$$

Damped Trend, Multiplicative Seasonality Model

$$X_t = (b_0 + b_1 \phi t) I_t + \varepsilon_t$$

Initial values

$$T_0 = \frac{m_k - m_1}{(k-1)p\phi}$$

$$S_0 = m_1 - \frac{p}{2} T_0 \phi$$

then

$$\hat{X}_0 = (S_0 + \phi T_0) I_{1-p}$$

$$S_t = S_{t-1} + \phi T_{t-1} + \alpha(2-\alpha) e_t / I_{t-p}$$

$$T_t = \phi T_{t-1} + \alpha(\alpha - \phi + 1) e_t / I_{t-p}$$

$$I_t = I_{t-p} + \delta[1 - \alpha(2-\alpha)] e_t / S_t$$

$$\hat{X}_t = (S_t + \phi T_t) I_{t-p+1}$$

References

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