CCF

Notation

The following notation is used throughout this chapter unless otherwise stated:

X, *Y*	Any two series of length *n*
$r_{xy}(k)$	Sample cross correlation coefficient at lag *k*
S_x	Standard deviation of series *X*
S_y	Standard deviation of series *Y*
$C_{xy}(k)$	Sample cross covariance at lag *k*

Cross Correlation

The cross correlation coefficient at lag *k* is estimated by

$$r_{xy}(k) = \frac{C_{xy}(k)}{S_x S_y}$$

where

$$C_{xy}(k) = \begin{cases} \frac{1}{n}\sum_{t=1}^{n-k}(x_t - \bar{x})(y_{t+k} - \bar{y}), & k = 0,1,2,\ldots \\ \frac{1}{n}\sum_{t=1}^{n+k}(y_t - \bar{y})(x_{t-k} - \bar{x}), & k = -1,-2,\ldots \end{cases}$$

$$S_x = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(x_t - \bar{x}\right)^2}$$

$$S_y = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(y_t - \bar{y}\right)^2}$$

The cross correlation function is not symmetric about $k = 0$.

Approximate standard error of $r_{xy}(k)$ is

$$se\left(r_{xy}(k)\right) \cong \sqrt{\frac{1}{n-|k|}}, \quad k = 0, \pm 1, \pm 2, \ldots$$

The standard error is based on the assumption that the series are not cross correlated and one of the series is white noise. (The general formula for the standard error can be found in Box and Jenkins, 1976, p. 376, 11.1.7.)

References

Box, G. E. P., and Jenkins, G. M. 1976. *Time series analysis*: *Forecasting and control*. San Francisco: Holden-Day.